OPTIMAL CONTROL PROBLEMS WITH INTERMEDIATE SINGULAR POINTS

EAST CHINA NORMAL UNIVERSITY

Student: Chernov Artem

Supervisor: Ni Ming Kang

Hefei
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Introduction

Control systems – are the systems that consist of basic parameters and additional parameter – control function.

For example, the rudder function while automobile moves. It effects trajectory and quality of moving. Also it is necessary to remark that trajectory of moving depends on some external effects, for example – weather conditions or road conditions.

All these effects should be considered in the equations of a system. The basic object of optimal control theory is to define and work out methods of control theory functions that help to obtain the best required result. It could be a minimization time process or income maximization (economics). The investigation of how small parameters effects on optimal control solution is an actual nowadays problem.

My area of scientific research interests includes the general theory of optimal control for linear and nonlinear systems of description and application of the basic principles in designing optimal controllers.

The objective is to minimize functional

$$J(u) = \frac{1}{2}\begin{pmatrix} x_1(t_1)-\xi_1 \\ x_2(t_1)-\xi_2 \end{pmatrix}' \begin{pmatrix} F_1(x_1(t_1)-\xi_1) \\ F_2(x_2(t_1)-\xi_2) \end{pmatrix} + \begin{pmatrix} x_1(t_1) \\ x_2(t_1) \end{pmatrix}' \begin{pmatrix} g \\ h \end{pmatrix} +$$

$$+\frac{1}{2}\sum_{i=1}^{2}\int_{t_{i-1}}^{t_i} \left(\begin{pmatrix} x_i(t) \\ u_i(t) \end{pmatrix}' \mathbb{W}_i(t) \begin{pmatrix} x_i(t) \\ u_i(t) \end{pmatrix} + \begin{pmatrix} d_i(t) \\ q_i(t) \end{pmatrix}' \begin{pmatrix} x_i(t) \\ u_i(t) \end{pmatrix} \right) dt$$

on the trajectories of the system

$$X'(t) = \begin{cases} A_1(t)x^-(t) + B_1(t)U(t) + f_1(t), 0 \le t \le t_1, \\ A_2(t)x^+(t) + B_2(t)U(t) + f_2(t), t_1 \le t \le T \end{cases}$$

$$x^-(0) = x^0$$

$$x^+(t_1) = x^-(t_1) + d$$

Asymptotic of optimal control problems solution

Lets find an asymptotic solution of the functional P_ε

$$J_\varepsilon(u_1,u_2) = \varepsilon\Big(\frac{1}{2}\langle x_1(t_1)-\xi_1, F_1(x_1(t_1)-\xi_1)\rangle + \\ +\frac{1}{2}\langle x_2(t_1)-\xi_2, F_2(x_2(t_1)-\xi_2)\rangle + \\ +\langle g, x_1(t_1)\rangle + \langle h, x_2(t_1)\rangle\Big) + \\ +\int_0^{t_1}\left(\frac{1}{2}\left\langle\begin{pmatrix} x_1(t)\\ u_1(t)\end{pmatrix}, \begin{pmatrix} W_1(t) & S_1(t)\\ S_1^*(t) & R_1(t)\end{pmatrix}\begin{pmatrix} x_1(t)\\ u_1(t)\end{pmatrix}\right\rangle + \\ +\left\langle\begin{pmatrix} d_1(t)\\ q_1(t)\end{pmatrix}, \begin{pmatrix} x_1(t)\\ u_1(t)\end{pmatrix}\right\rangle\right)dt + \\ +\int_{t_1}^{T}\left(\frac{1}{2}\left\langle\begin{pmatrix} x_2(t)\\ u_2(t)\end{pmatrix}, \begin{pmatrix} W_2(t) & S_2(t)\\ S_2^*(t) & R_2(t)\end{pmatrix}\begin{pmatrix} x_2(t)\\ u_2(t)\end{pmatrix}\right\rangle + \\ +\left\langle\begin{pmatrix} d_2(t)\\ q_2(t)\end{pmatrix}, \begin{pmatrix} x_2(t)\\ u_2(t)\end{pmatrix}\right\rangle\right)dt \to min \tag{1}$$

on the trajectories of the system

$$X'(t) = \begin{cases} A_1(t)x^-(t) + B_1(t)U(t) + f_1(t), 0 \le t \le t_1, \\ A_2(t)x^+(t) + B_2(t)U(t) + f_2(t), t_1 \le t \le T \end{cases} \quad (2)$$

$$x^-(0) = x^0$$

$$x^+(t_1) = x^-(t_1) + d$$

Here $\varepsilon > 0$ small parameter, other restrictions are the same as earlier. Functions, which form the solution of perturbed problem (1) could be found in the view of these series:

$$u_j(t,\varepsilon) = \sum_{i \ge 0} \varepsilon^i u_{ij}(t)$$

$$x_j(t,\varepsilon) = \sum_{i \ge 0} \varepsilon^i u_{ij}(t) \text{ , j=1,2}$$

Wich are substituted in (2), after that we make an equation, using the coefficients of ε with the same power indexes. The functional (1) that we are minimizing now could be written in the form

$$J_\varepsilon(u_1, u_2) = \sum_{i \geq 0} \varepsilon^i J_i$$

Illustrative example of solution the problem

$$J = \frac{1}{2}(x_1(1) - 1)^2 + \frac{1}{2}(x_2(1) - 3)^2 + \int_0^1 u_1^2 dt + \int_1^2 u_2^2 dt \to min$$

$$\begin{cases} \dfrac{dx_1}{dt} = u_1 \\ \dfrac{dx_2}{dt} = u_2 \\ x_1(0) = 0 \\ x_2(1) = x_1(1) + 7 \end{cases}$$

Find optimal control

$$u_1(t) = \psi_1, \quad 0 \le t \le 1$$

$$u_2(t) = \psi_2, \quad 1 \le t \le 2$$

$$\frac{d\psi_1}{dt} = 0 \quad \to \quad \psi_1 = C_1, \quad 0 \le t \le 1$$

$$\frac{d\psi_2}{dt} = 0 \ \to \quad \psi_2 = C_2;; \quad \psi_2 = 0 \ \to \ C_2 = 0, \quad 1 \le t \le 2$$

$$\psi_2(1) - \psi_1(1) = (x_2(1) - 3) + (x_1(1) - 1)$$

$$u_2 = \psi_2 = 0, \quad 1 \le t \le 2$$

$$\frac{dx_2}{dt} = 0 \rightarrow \ x_2 = const = K$$

$$u_1 = \ \psi_1 = C_1$$

$$\frac{dx_1}{dt} = C_1; \ x_1(t) = C_1 t + M$$

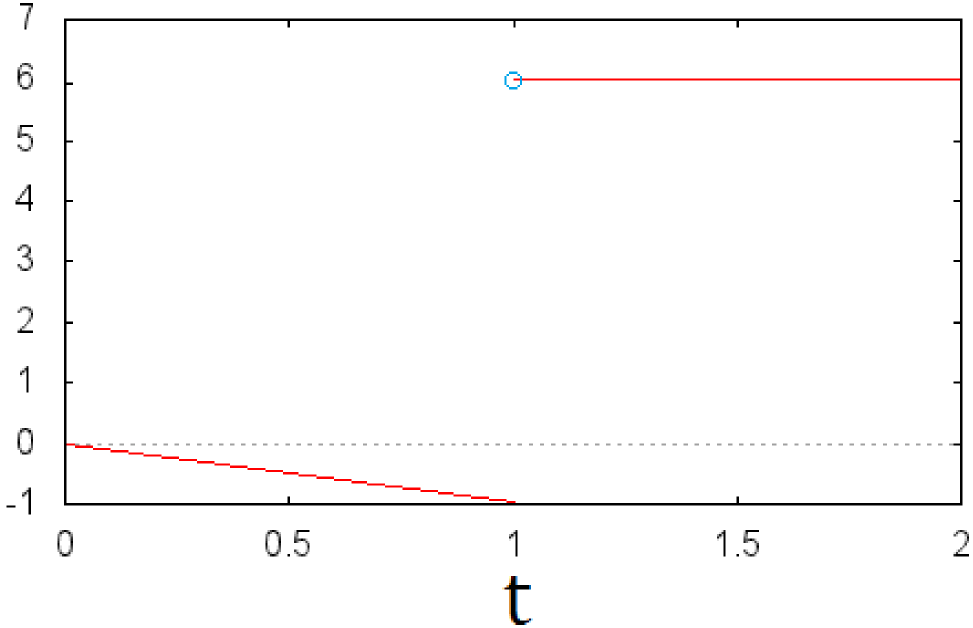
$$x_1(0) = C_1 \cdot 0 + M = M = 0 \rightarrow \ x_1(t) = C_1 t$$

$$x_2(1) = K = C_1 + 7$$

$$-C_1 = (C_1 + 7 - 3) + (C_1 - 1); \ \ C_1 = -1$$

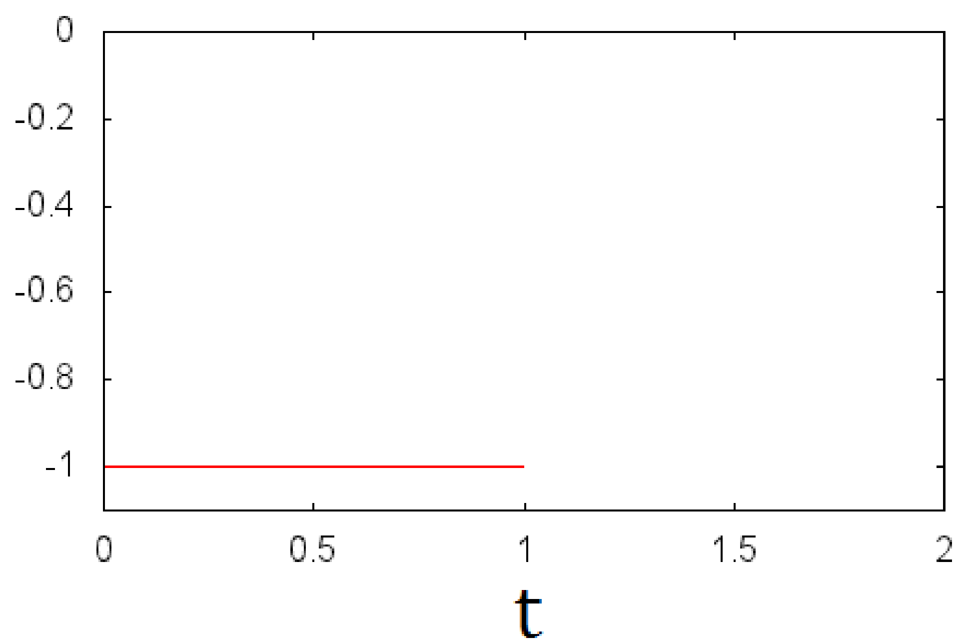
$$u^* = \begin{cases} -1, & 0 \leq t \leq 1 \\ 0, & 1 \leq t \leq 2 \end{cases}$$

$$x^* = \begin{cases} -t, & 0 \leq t \leq 1 \\ 6, & 1 \leq t \leq 2 \end{cases}$$



trajectory

control



Numerical experiment

$$J_\varepsilon = \varepsilon((x_1(1)+1)^2 + (x_2(1)-2)^2) + \frac{1}{2}\int_0^1 (x_1^2 + u_1^2)\, dt + \frac{1}{2}\int_1^2 u_2^2\, dt$$

on the trajectories of the system

$$\begin{cases} \dfrac{dx_1}{dt} = u_1 \\ x_1(0) = 0 \\ \dfrac{dx_2}{dt} = x_2 + u_2 \\ x_2(1) = x_1(1) + 3 \end{cases}$$

If $\varepsilon = 0.1$ **we have**

$$\begin{cases} -S_1^* x_1 + B_1^* \psi_1 - R_1 u_1 - q_1 = 0, \\ \psi_1 - u_1 = 0 \Rightarrow \psi_1 = u_1 \\ \dfrac{d\psi_1}{dt} = W_1 x_1 - A_1^* \psi_1 + S_1 u_1 + d_1 \Rightarrow \dfrac{d\psi_1}{dt} = x_1 \\ \dfrac{dx_1}{dt} = u_1 = \psi_1 \Rightarrow \dfrac{dx_1}{dt} = \dfrac{d\psi_1}{dt} = x_1 \Rightarrow \dfrac{dx_1}{dt} - x_1 = 0 \\ \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \\ x_1(t) = C_1 e^t + C_2 e^{-t} \\ x_1(0) = C_1 + C_2 = 0 \Rightarrow C_2 = -C_1 \\ x_1(t) = C_1 e^t - C_1 e^{-t} \\ \dfrac{d\psi_1}{dt} = C_1 e^t - C_1 e^{-t} \Rightarrow \psi_1(t) = C_1 e^t + C_1 e^{-t} \Rightarrow u_1(t) = C_1 e^t + C_1 e^{-t} \end{cases}$$

Similarly we have

$$\begin{cases} -S_2^* x_2 + B_2^* \psi_2 - R_2 u_2 - q_2 = 0 \Rightarrow \psi_2 = u_2 \\ \dfrac{d\psi_2}{dt} = W_2 x_2 - A_2^* \psi_2 + S_2 u_2 + d_2 \Rightarrow \dfrac{d\psi_2}{dt} = -\psi_2 \Rightarrow \psi_2(t) = Ce^{-t} = u_2(t) \\ \dfrac{dx_2}{dt} = x_2 + Ce^{-t} \Rightarrow \end{cases}$$

$$\frac{dx_2}{dt} = x_2 \Rightarrow x_2(t) = Ae^t$$

As a result

$$x_2(t) = \frac{3.8e^2 + 2.2}{e(1.4e^2 + 0.6)} \cdot e^t, t > 1$$

$$x_1(t) = \frac{-0.4e}{1.4e^2 + 0.6}(e^t - e^{-t}), t < 1$$

$$u_1(t) = \frac{-0.4e}{1.4e^2 + 0.6}(e^t + e^{-t}), t < 1$$

$$u_2(t) = 0, t > 1.$$

Thus, the functional

$$J_\varepsilon(u_1,u_2)=0.1\cdot(\frac{-0.4e}{1.4e^2+0.6}(e-e^{-1})+1)^2+0.1\cdot(\frac{3.8e^2+2.2}{e(1.4e^2+0.6)}\cdot e-2)^2+$$

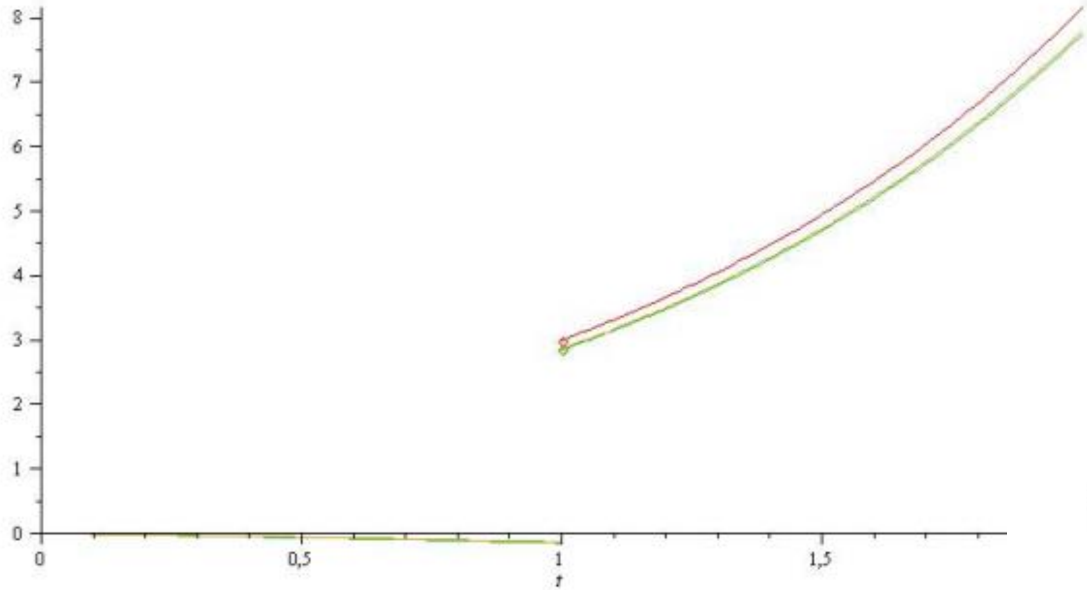
$$+\frac{1}{2}\int\limits_0^1(x_1^2+u_1^2)dt\approx 0.153299$$

If $\varepsilon=0.1$

$$J_\varepsilon(u_{10},u_{20})=0.2$$

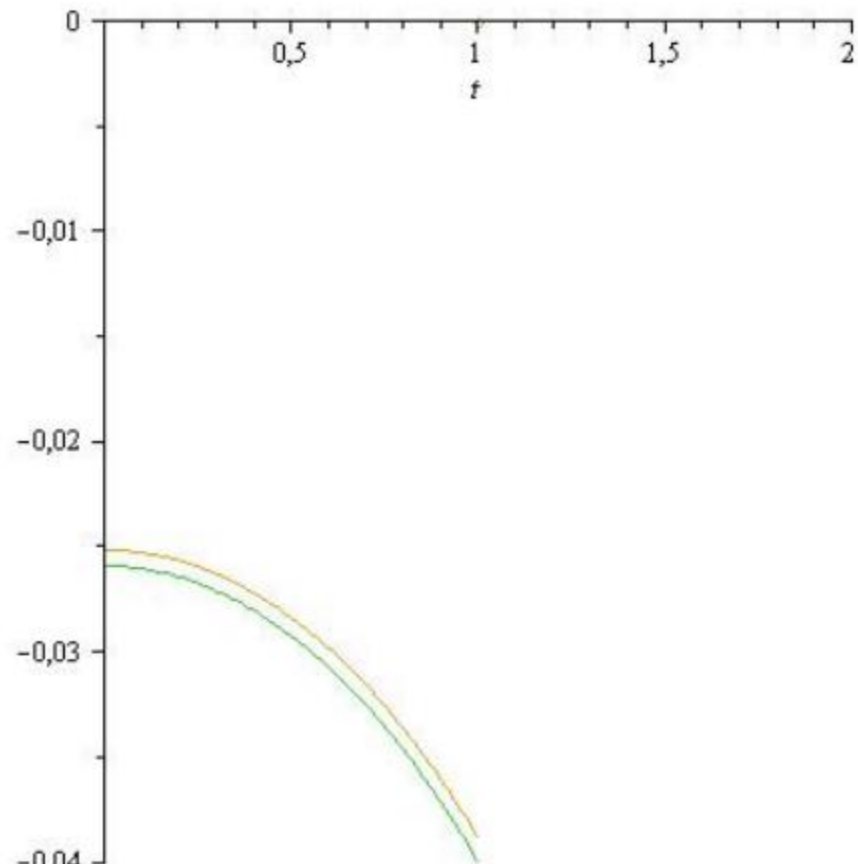
$$J_\varepsilon(\widetilde{u}_{11},\widetilde{u}_{21})=0.157633$$

$$J_\varepsilon(u_1,u_2)=0.153299$$



trajectory

control



Conclusion

In this article linear-quadratic problem of optimal control was considered. In this problem the functional that was minimized depends on the state variable values in some intermediate points. Trajectories and control are discontinuous in switch points. In this report was considered the case when function trajectory jump in fixed point, is known and value of state variable in initial moment is known.

Now the optimal control theory has a period of rapid development in connection with presence of difficult and interesting mathematical problems, with lots of different applications integrated in such areas as economics, biology, medicine, nuclear energy and some others.

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Thank you for your time